

Study of the Basic Parameters Characterized the (D-T) Hot Plasma Fusion Reaction In General Dense Plasma Focus Devices

Raad Hameed Majeed

Department of Physics, University of Baghdad,
College of Education (Ibn al-Haitham),

Abstract

Plasma focus devices can be considered (PFD) as a strong source for the elementary charge particles, neutrons, protons, and soft x-rays are the main reason for its importance in many experimental applications. The dependency of the basically hot plasma parameters such as reactivity, reaction rate, and energy of the emitted neutrons, upon the total cross section of D-T fusion reaction still up to now the essential factor governed the primary calculations, and accuracy, for both experimental and theoretical studies. Formulism describing the physical behavior of a given experiment still varies from others experiments and indeed, a theoretical modification for the empirical formulas plays a rule to achieve compatible results.

Keywords: plasma focus, neutron yield, hot plasma parameters, d-t reaction, total cross section, fusion power.

رعد حميد مجيد

خلاصة

تعزى كثرة التطبيقات العملية لاجهزة بؤرة البلازما الكثيفة الى خصوصية هذه الاجهزة باعتبارها مصدر للجسيمات الاولية المشحونة (النيوترونات والبروتونات) اضافة الى الاشعة السينية. اعتمادية العوامل الاساسية للبلازما الحارة كالفعاية، معدل التفاعل، وطاقة النيوترونات المنبعثة على المقطع العرضي الكلي للتفاعل لاتزال حتى وقتنا الحاضر من اهم العوامل المؤثرة والمحكمة لدقة الحسابات النظرية والعملية وعملية التوافق مابينهما. العلاقات التجريبية التي توصف سلوك المقطع العرضي الكلي تختلف من تجربة الى اخرى وبالتالي وجد من الضروري اجراء تحوير او تطوير لمثل هذه العلاقات كي يتم البلوغ الى توافق معقول بين النتائج النظرية والعملية.

Introduction

The relevance and importance of a plasma focus devices (PFD), as intense source of neutrons are due to essentially the low costs and dimensions of the device itself, to its high performance in term of neutrons emitted per discharge and to the type of spectra of the emitted neutrons. For instance a D-T filled advance plasma focus (APF) with capacitor bank energy of about 200 KJ can give about 10^{14} neutrons per discharge, these being emitted almost point wisely and almost mono energetically at about 14 MeV [1].

Plasma focus devices operated with D-T fuels suitable to generate a neutron yield about $Y_n = 10^{15}$, during a 1-year run, an overall fluencies affecting materials to the order of 0.1 to 1.0 displacement per atom (DPA) (1 DPA is equal to a mean neutrons flux of 4.5×10^{16} neutron $m^{-2}s^{-1}$ for 1 year) for such testing purposes, at a very low cost relative to other methods currently being considered [2].

The main technological problems are indeed related to the stability of the discharge repetition rates, which nowadays can reach 10 – 15 Hz and

can be sustained for times of the order of magnitude of the hour. If the technology related to these performances could be improved somehow, PF devices will be highly competitive as neutrons sources in the future. Therefore, great theoretical and experimental efforts should be directed towards a better characterization of the energetic and angular distribution of the emitted neutrons, in other words towards a better characterization of the neutronics of PF machines. This means that the fusion doubly-differential cross sections are to be more correctly evaluated, and the models giving the corresponding fusion reaction rates more precisely set-up, as well as the description of the plasma dynamics in the pinch zone of the PF [1].

Generally, several parameters like the initial pressure, insulator length [3], and capacitor bank energy [4], are effective to the various products of the PF device. However, the results are collected in an important factor called the drive parameter, which is defined as $S = \left(\frac{I}{a}\right) / \sqrt[2]{P}$, in which I, a, P is the maximum current flowing into the pinch, anode radius, and the initial filling gas pressure at the optimized neutron regime, respectively [5]

Theoretical models

Presently two main classes of models for the evaluation of fusion reaction rates are used, namely.

- 1- The "BEAM-TARGET" MODEL
- 2- The "MAXWELLAIN REACTIVITY". MODEL

The beam target model was introduced by Bernstein and Comisar [6] in the late '60s to early of '70s. It is presently the most largely accepted model. This model assumes that the colliding ions in the pinch zone of the PF can be divided into two sub-populations of "beam ions" and "target ions". The target ions are assumed in many calculations to be at rest; axially only moving target ions have also been considered. The fusion reactions are assumed to happen in so a small zone called the pinch region. Reaction rates are then calculated as being the production term of the collision of the

Boltzmann equation for the interaction of the two sub-populations. In doing this, one has to know the energetic and angular dependence of the beam ions distribution functions. Bernstein and Comisar were the first to perform beam-target reactivity calculations assuming energetic and angular dependence for the beam ions distribution functions which were dictated by a few experimental studies [6, 7].

In more recent years there have been a lot of efforts for both in the direction of theoretical and experimental determine the ions, neutrons beams distribution function. The consistence of these results has been checked using the beam-target model, and considerable improvements have made since the times of Bernstein and Comisar [1].

By definition the reaction rate R for reaction i in a space independent problem is given by [1].

$$R = n_D n_T \iiint f(\vec{v}_D) F(\vec{v}_T) g \frac{d\sigma_i}{d\Omega} d\Omega d\vec{v}_D d\vec{v}_T \quad (1)$$

Where n_D and n_T are the beam and target ions densities with unit-normalized distribution functions f and F respectively; g is the modulus of the relative velocity between beam and target ions and $\frac{d\sigma_i}{d\Omega}$ is the doubly differential cross section.

In the Bernstein – Comisar formalism one can introduce the dummy variable E by use of a Dirac Delta distribution, so that

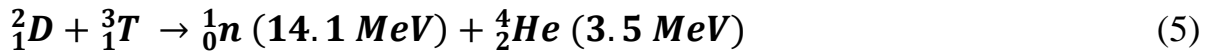
$$R = n_D n_T \iiint f(\vec{v}_D) F(\vec{v}_T) g \frac{d\sigma_i}{d\Omega} \delta(E - E_n) d\Omega d\vec{v}_D d\vec{v}_T dE \quad (2)$$

Where E_n is the emitted neutron energy in the case of axially moving target ions. By the laws of energy and momentum conservation, and from the general basic fundamental equation of calculating the energy for the charged particles from any nuclear reaction given below [8].

$$\sqrt[2]{E_3} = v \pm \sqrt[2]{v^2 + \omega} \quad (3)$$

$$\text{Where } v = \frac{\sqrt[2]{M_1 M_3 E_1}}{M_3 + M_4} \cos \theta \quad \text{and} \quad \omega = \frac{M_4 Q + E_1 (M_4 - M_1)}{M_3 + M_4} \quad (4)$$

For the case of the binary D-T reaction,



Where

$$E_3 = E_n, M_1 = M_d, M_2 = M_t, M_3 = M_n, M_4 = M_{He}, E_1 = E_d$$

And $Q_{value} = 17.6 \text{ MeV}$ is the Q-value of the above D-T fusion reaction, E_d is the deuteron bombarding energy.

Substituting the values for the quantities $v, \omega, M_1, M_2, M_3, M_4, E_1, Q$ as it is described above in equation (3), and taken into account some mathematical analysis steps to get or deduced a formula for evaluating the energy of the emitted neutrons that given below.

$$E_n = \frac{20 Q + 12 E_d}{25} \left[\sqrt[2]{1 - \gamma^2 \sin^2 \theta} + \gamma \cos \theta \right]^2 \quad (6)$$

$$\text{Where } \gamma^2 = \frac{2 E_d}{20 Q + 12 E_d} \quad (7)$$

Equation (5), explain the relationship between the energy of the emitted neutrons from the D-T fusion reaction with the energy of bombarding deuterons and reaction angle E_d, θ , respectively.

A most important quantity for the analysis of nuclear reaction is the *cross section*, which measure the probability for the occurrence of a nuclear reaction. The cross section $\sigma_{12}(v_1)$ is defined as the number of reactions per target nucleus per unit time

when the target is hit by a unit flux of projectile particles that is by one particle per unit target area per unit time. Actually, the above definition applies in general to particles with relative velocity v , and is therefore symmetric in the two particles, since we have $\sigma_{12}(v) = \sigma_{21}(v)$.

Cross section can also express in terms of the center of mass energy, and we have $\sigma_{12}(\epsilon) = \sigma_{21}(\epsilon)$. In most cases, however, the cross section are measured in experiments in which a beam of particles with energy ϵ , measured in the laboratory frame, hits a target at rest. The corresponding beam-target cross section $\sigma_{12}^{bt}(\epsilon_1)$ is related to the center-of-mass cross section $\sigma_{12}(\epsilon)$ by

$$\sigma_{12}(\epsilon) = \sigma_{12}^{bt}(\epsilon_1) \tag{8}$$

With $\epsilon_1 = \epsilon \cdot (m_1 + m_2)/m_2$

If the target nuclei have density n and are at rest or all move with the same velocity, and the relative velocity is the same for all pairs of projectile-target nuclei, then the probability of reaction per unit times is obtained by multiplying the probability per unit path times the distance travelled in the unit time, which gives $n_2 \sigma(v)v$.

Another important quantity is the **reactivity**, which defined as the probability of reaction per unit time per unit density of target nuclei. It is just given by the product σv . In general, target nuclei moves, so that the relative velocity v is different for each pair of interacting nuclei. In this case, we compute an **averaged reactivity**.

$$\langle \sigma v \rangle = \int \sigma(v)v f(v)dv, \tag{9}$$

Where $f(v)$ is the distribution function of the relative velocities, normalized in such a way that $\int_0^\infty f(v)dv = 1$.

Both controlled fusion fuels and stellar media are usually mixtures of elements where species `1` and `2` have number densities n_1, n_2 , respectively. the volumetric reaction rate, that is the number of reactions per unit time and per unit volume is then given by

$$R = \frac{n_1 n_2}{1+\delta_{12}} \langle \sigma v \rangle = \frac{f_1 f_2}{1+\delta_{12}} n^2 \langle \sigma v \rangle \tag{10}$$

Where n is the total nuclei number density, f_1 and f_2 , are the atomic fraction of species "1" and "2", respectively. The Kronecker symbol δ_{12} (with $\delta_{12} = 1, if i = j$ and $\delta_{12} = 0$ elsewhere) is introduced to properly take into account the case of reaction between like particles. Equation 7 show a very important feature for fusion energy research: the volumetric reaction rate is proportional to the square of the density of the mixture. For feature reference, it is also useful to recast it in terms of the mass density ρ of the reacting fuel

$$R_{12} = \frac{f_1 f_2}{1 + \delta_{12}} \frac{\rho^2}{\bar{m}^2} \langle \sigma v \rangle \quad (11)$$

Where \bar{m} is the average nuclear mass.

Here, the mass density is computed as $= \sum_j n_j m_j = n \bar{m}$, where the sum is over all species. We also immediately see that the specific reaction rate is proportional to the mass density, again the role of the density of the fuel is achieving efficient release of fusion energy [8].

Main controlled fusion reaction

The most important main controlled fusion reactions necessary as power source applications that recently used in many interested countries which supported huge efforts are listed in table (1).

Table 1. Main controlled fusion reactions [9].

Reaction	$\sigma(10 \text{ keV})$ barn	$\sigma(100 \text{ keV})$ barn	σ_{\max} barn	ϵ_{\max} keV
$D + T \rightarrow \alpha + n$	2.72×10^{-2}	3.43	5.0	64
$D + D \rightarrow T + p$	2.81×10^{-4}	3.3×10^{-2}	0.096	1250
$D + D \rightarrow {}^3\text{He} + n$	2.78×10^{-4}	3.7×10^{-2}	0.11	1750
$T + T \rightarrow \alpha + 2n$	7.90×10^{-4}	3.4×10^{-2}	0.16	1000
$D + {}^3\text{He} \rightarrow \alpha + p$	2.20×10^{-7}	0.1	0.9	250

In our work, we concentrate on study the reaction between the hydrogen isotopes, deuterium and tritium, which are the most important fuels for controlled fusion research, namely, the D-T reaction.



The D-T reaction has the largest cross section, which reaches its maximum (about 5 barns) at the relatively modest energy of 64 keV (see fig. 1). Its $Q = 17.6 \text{ MeV}$ is the largest of this family of reactions. It is to be observed that the cross section of this reaction is characterized by a broad resonance for the formation of the compound ${}^5\text{He}$ nucleus at $\epsilon \cong 64 \text{ keV}$ [9].

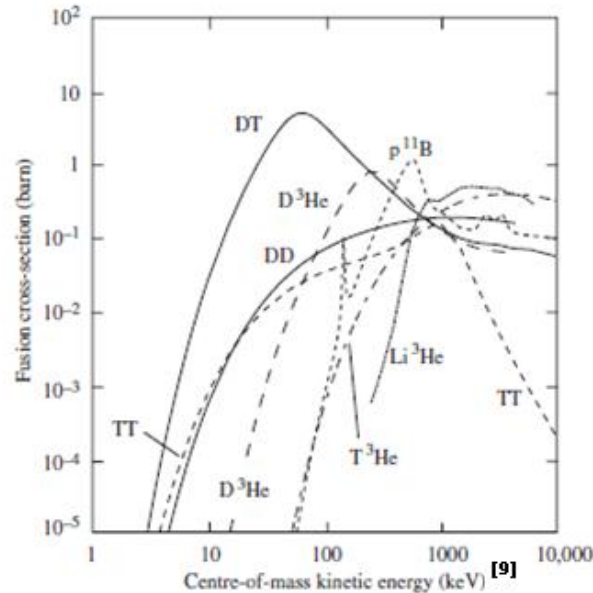


Figure 1. Fusion cross sections for the main controlled fusion reactions

Calculation and Results

The calculations are concentrated on the D-T fusion reaction, because of its huge applications in power source due to their high energy release, such as fusion reactors, (Tokomak), and others small systems like the dense plasma focus devices (DPF).

Clearly, as it is described previously, in order to calculate such hot parameters, i.e., reactivity, reaction rate, and the energy of emitted neutrons are all controlled by the cross section, which represents the essential factor in the calculations. A widely used parameterization of fusion reaction cross section is [9]

$$\sigma \approx \sigma_{goem} \times \mathcal{T} \times \mathcal{R} \quad (12)$$

Where σ_{goem} is a geometrical cross section, \mathcal{T} is the barrier transparency, and \mathcal{R} is the probability that nuclei come into contact fuse. The first quantity is of the order of the square of the de-Broglie wavelength of the system:

$$\sigma_{goem} \approx \lambda^2 = \left(\frac{\hbar}{m_r v} \right)^2 \propto \frac{1}{\epsilon}, \quad (13)$$

Where \hbar is the reduced Planck constant and m_r is the reduced mass. Equation 9 concerning the barrier transparency, and its often well approximated by [9].

$$\mathcal{T} \cong \exp\left(-2\sqrt{\frac{\epsilon}{\epsilon_G}}\right) \quad (14)$$

Which is known as the Gamow factor, where

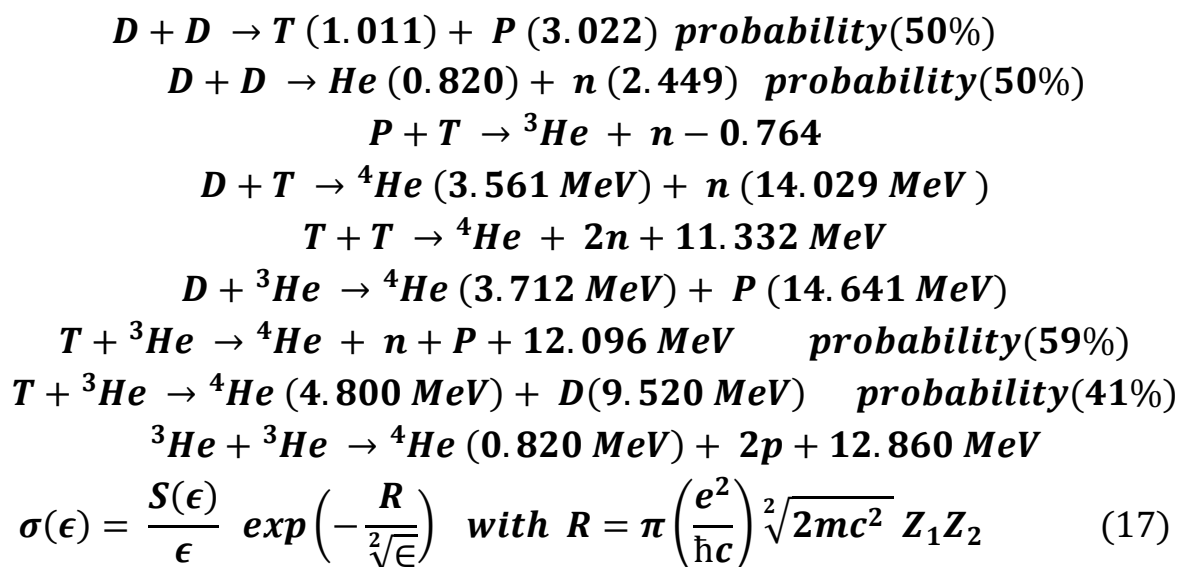
$$\epsilon_G = (\pi\alpha_f Z_1 Z_2) 2m_r c^2 = 986.1 Z_1^2 Z_2^2 A_r \text{ keV} \quad (15)$$

Is the Gamon energy, $\alpha_f = \frac{e}{\hbar c} = \frac{1}{137.04}$, is the fine structure constant commonly used in quantum mechanics, $A_r = m_r/m_p$

The reaction characteristics \mathcal{R} contains essentially all the nuclear information's of the specific reaction. it takes substantially different values depending on the nature of the interaction characterizing the reaction. It is largest for reaction due to strong nuclear interactions; it is smaller by several orders of magnitude for electromagnetic nuclear interactions; and still smaller by as many as 20 orders of magnitude for weak interactions. For most reactions, the variation of $\mathcal{R}(\epsilon)$ is small compared to the strong variation due to the Gamon factor. In conclusion the cross section is often written as [9].

$$\sigma(\epsilon) = \frac{S(\epsilon)}{\epsilon} \exp\left(-2\sqrt{\frac{\epsilon}{\epsilon_G}}\right) \quad (16)$$

Where the function $S(\epsilon)$ is called the astrophysical S factor, which for many important reactions is a weakly varying function of the energy [9]. Plasma reactivity calculations require reaction cross sections for energies well below those at which direct measurement are practicable [10]. A more convents or suitable formula for calculation the total cross section for D-T or others main controlled fusion reactions (listed below), which has a compatible agreement results with the really experimental results is given by.



Where the cross section is expressed in centre of mass units, $\epsilon = \frac{1}{2}mv^2$, $m = m_1m_2/(m_1 + m_2)$ and v is the relative velocity of the interacting particles which have masses m_1 and m_2 and charges Z_1 and Z_2 respectively. the constants e , \hbar and c have their usual meaning. $S(\epsilon) = A \exp(-\beta \epsilon)$ and the parameters A , β and R are given in table (2). Note that laboratory energies may be used if the substitution $\epsilon = \left(\frac{m}{m_1}\right) \epsilon_{lab}$ has to be considered [10].

Table 2. Low energy cross section parameterization [10].

Reaction	A (barns - keV)	β (keV ⁻¹)	R (keV ^{1/2})
$D - D_p$	52.6	-5.8×10^{-3}	31.39
$D - D_n$	52.6	-5.8×10^{-3}	31.39
$D - T$	9821	-2.9×10^{-2}	34.37
$T - T$	175	9.6×10^{-3}	38.41
$D - {}^3He$	5666	-5.1×10^{-3}	68.74
$T - {}^3He$	2422	4.5×10^{-3}	76.82
${}^3He - {}^3He$	5500	-5.6×10^{-3}	153.70

By testing equation 14, for the D-T fusion reaction and in order to arrive results gives high agreement with the corresponding experimental published results, we find that it

is very necessary to introduce a correction factor related to each deuterons energy (ϵ) in the above equation, and these case are completely described in table 3.

Theoretically, the energy of the emitted neutrons from the D-T fusion reaction can be exactly determined from equation (6) as a function of both the incident deuteron energy E_d and the reaction angle θ . The calculated results are completely described in Table 4.

Finally, the reactivity as a function of the temperature, obtained by numerical integration of the following equation with the best available cross section for the reactions of interest to controlled fusion.

$$\langle \sigma v \rangle = \left[\left(\frac{m_1 + m_2}{2\pi k_B T} \right)^{3/2} \int dV_c \exp \left(-\frac{m_1 + m_2}{2k_B T} V_c^2 \right) \right] \times \left(\left(\frac{m_r}{2\pi k_B T} \right)^{3/2} \int dV_c \exp \left(-\frac{m_r}{2k_B T} V^2 \right) \sigma(v) v \right)$$

The term in square bracket is unity, being the integral of a normalized Maxwellian, and we are left the integral over the relative velocity. By writing the volume element in velocity space as $= 4\pi v^2 dv$, and using the definition of center of mass energy ϵ , we finally get

$$\langle \sigma v \rangle = \frac{4\pi}{(2\pi m_r)^{1/2}} \frac{1}{(k_B T)^{3/2}} \int_0^\infty \sigma(\epsilon) \epsilon \exp(-\epsilon/k_B T) d$$

For the D-T fusion reaction, which is by far the most important one for present fusion research, the following expression is used to calculate the reactivity [8]. And the present calculated results are completely described in figure 3.

$$\langle \sigma v \rangle = 9.10 \times 10^{-16} \exp \left(-0.572 \left| \ln \frac{T}{64.2} \right|^{2.13} \right) \text{ cm}^3/\text{s} \quad (18)$$

Discussion and Conclusion

Clearly, from the results about the total cross section for the D-T fusion reaction calculated by equation (17), it is appear a common shift from the published experimental results and one can interpreted that by some physical reasons that directly correlated with the fundamentals parameters deal with the system or device designing, geometrical dimension for the cathode and anode, and the operating factors, such as the fuel pressure, initial power, and we can added another reason deal with the construction time for building the experiment, in which that any system are exactly differ in all covering physical conditions with the recent ones. In other words, it is necessary to available a given empirical formula for each systems (experimental devices). Therefore, we concluded that it is important and necessary to modify the formula for the total cross section for the D-T fusion reaction by introducing a fixed correction factor for certain deuteron energy/or energy intervals to avoid the disagreement between the theoretical and experimental results.

From Table 1 and Table 3, we see that our calculated results about the

total reaction cross section, after introducing the correction factors are more compatible with the published results [9]. Also, from Figure 3, the calculated results about the D-T fusion reaction reactivity by using equation (18) give or reflect a physical behavior that are more suitable with the corresponding published results, and this case can be interpreted for the reason of the cross section data are available in the reactivates equation. Finally it's useful to suggest the recommendation of the modified formulas instead of the previously described ones to be applied in the recently systems.

From Table 4, it is clear that the energies of emitted neutron, which calculated by our expressed formula at incident reaction angle of 90 degree are of quite agreement with the recommended value of (14.029 MeV), and this case can be interpreted as, there exist a small percentage of incident deuterons are scattered from its original direction, and all the really occurring physical experimental phenomenon's can be explained at this angle instead of others angles.

References

1. Federico Rocchi and Marco Sumini, "Characterization of neutron emission from a D-D filled plasma focus", M&C 2001 Salt Lake City, Utah, USA, September 2001 .
2. Sor Heoh Saw and Sing Lee, "Scaling the plasma focus for fusion energy considerations", Int. J. Energy Res., vol. 35. Pp.81-88,2011.
3. M. Zakaullah, T.J. Baig, S. Beg, and G. Murtaza, "Effect of insulator sleeve length on neutron emission in a plasma focus", Phys. Let. A, Vol.137, no. 1/2, pp. 39-43, may 1989.
4. A. Talaei and S. M. Sadat Kiai, "Study the influence of the bank energy on the dynamical pinch in plasma focus', J. Fusion energy, vol.28, no. 3, pp. 304-313, sep. 2009.
5. T. Zhang, R. S. Rawat, S. M. Hassan , J. J. Lin, S. Mahmood, T. L. Tan, S. V. Springham, V. A. Gribkov, P. Lee, and S. Lee, "Drive parameters as a design consideration for Mather and Fillippov types of plasma focus,"IEEE Tran. Plasma Sci., vol. 34, no. 5, pp. 2356-2365, oct. 2006.
6. Bernstein, M.J, Comisar, G.G., "Neutron energy and flux distribution from a crossed-field acceleration model of plasma focus and Z-pinch discharges",the physic of fluids, 15, 4, pp 700-707, 1972.
7. Tiseanu, I., Mandache, N., Zambreanu, V., "Energetic and angular distribution of the reacting deuterons in a plasma focus", plasma phys. Control. Fusion, Vol. 36, pp417-432, 1994.
8. Robly D.Evans , "The atomic nucleus", Mcgraw-Hill book company, inc., 1955.
9. Stefano Atzeni, Juroen M., " The physics of inertial ", Oxford University press, 2004.
10. O.N. Jarvis, www.Kayelaby.npl.co.uk, "Table of physics and chemical constants", Chapter 4, section 4.7.4, 2011.
11. T.A. Heitemes, G.A. Moses, J.F. Santarius, "Anaylsis of an improved fusion reaction rate model for use in fusion plasma simulation", fusion technology

institute, university of Wisconsin, 1500 engineering drive, Madison, WI 53706, UWFDM- Report, 2005.

Table 3. The recommended correction factors necessarily for calculations the D-T fusion reaction cross section.

Energy ϵ keV	Correction factor	Cross section $\sigma(\epsilon)$ barns
5	1.080	5.1779×10^{-4}
10	1.081	2.7020×10^{-2}
20	0.50	0.2015
30	0.35	0.5148
40	0.30	1.0253
50	0.32	2.0754
60	0.31	3.4198
64	0.375	5.0144
70	0.255	4.4785
80	0.150	4.0166
90	0.085	3.3684
100	0.0598	3.3427

Table 4. The calculated energy of emitted neutrons as a function of the reaction angle.

E_n MeV	$\theta = 0$ deg	$\theta = 45$ deg	$\theta = 60$ deg	$\theta = 90$ deg			
	E_d keV	E_n MeV	E_d keV	E_n MeV	E_d keV	E_n MeV	E_d keV
14.0842	10	14.2979	10	14.2350	10	14.1906	10
14.0923	20	14.3915	20	14.3020	20	14.2391	20
14.0963	30	14.4646	30	14.3546	30	14.2773	30
14.1004	40	14.5272	40	14.3997	40	14.3102	40
14.1044	50	14.5830	50	14.4400	50	14.3397	50
14.1084	60	14.6341	60	14.4769	60	14.3668	60
14.1125	70	14.6815	70	14.5113	70	14.3922	70
14.1165	80	14.7260	80	14.5436	80	14.4160	80
14.1205	90	14.7682	90	14.5743	90	14.4387	90
14.1246	100	14.8084	100	14.6036	100	14.4604	100
14.1286	110	14.8469	110	14.6317	110	14.4813	110

14.1326	120	14.8840	120	14.6587	120	14.5015	120
14.1366	130	14.9198	130	14.6849	130	14.5210	130
14.1407	140	14.9545	140	14.7103	140	14.5399	140
	150	14.9882	150	14.7349	150	14.5584	150

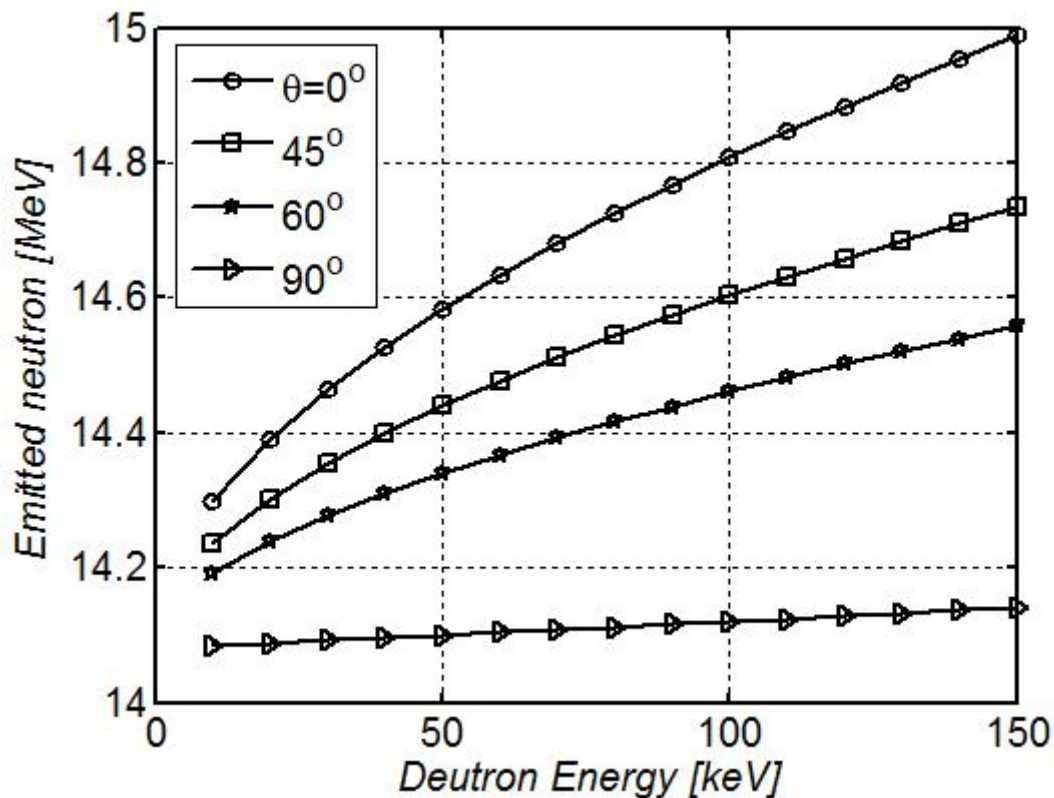


Figure 2. Variation of the emitted neutron energy versus the incident deuteron energy

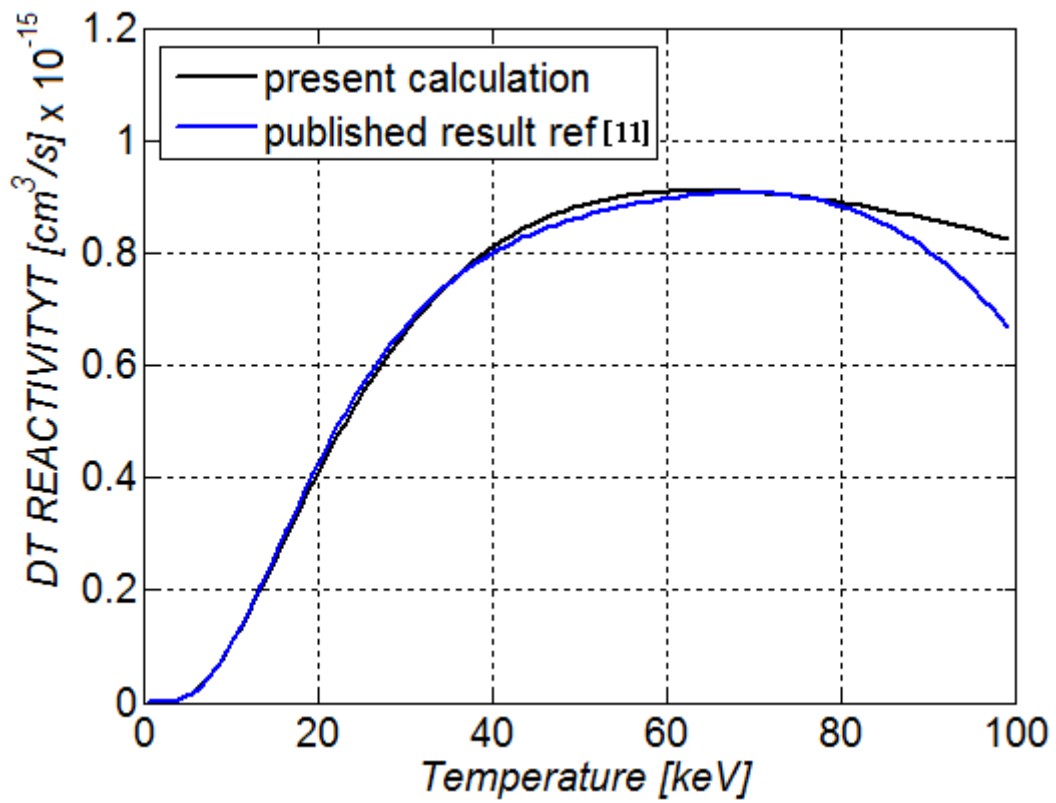


Figure 3. Variation of the D-T Reactivity versus the incident deuteron temperature