

## Schauder of (Fixed point) Theories in “Intuitionistic Fuzzy Metric Space” by (Occasionally Weakly Compatible) of Self Mapping

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### Abstract

The present paper is aimed to obtain some fixed point (FP) theories (Schauder) in an intuitionistic fuzzy metric space (IFMS) under using the ideas of four self-mapping, such that by Occasionally Weakly Compatible (OWC) self-mapping. As an application the existence of Schauder and illustrative by some example of (OWC) self-mapping in (IFMS) usage one process of fixed point theories Schauder will be studied using implicit relations. This work may be considered as a general statement of fuzzy metric space (FMS) and to show another sort from fixed point theories in (OWC) self-mapping which are studied by other researchers and unify many results in the topic of fixed point.

**Keywords:** Fuzzy Metric Space, Intuitionistic fuzzy metric space, Fixed point by Schauder four in Weakly Compatible Maps & Occasionally Weakly Compatible-Maps. theories

## مبرهنات شاورد النقطة الصامدة في الفضاء المترى الضبابي الحدسي بواسطة التخطيط البياني المتوافق الضعيف احيانا

اماني النقات كاظم

قسم/هندسة تقنيات ميكانيك القوى / فرع السيارات / الكلية التقنية الهندسية بغداد / الجامعة التقنية الوسطى - بغداد - العراق

### الخلاصة

يهدف هذا البحث للحصول على بعض نظريات النقطة الصامدة في الفضاء المترى الضبابي الحدسي باستخدام افكار التخطيط البياني لاربعة بواسطة التخطيط البياني المتوافق الضعيف احيانا كتطبيق على وجود شاورد والتوضيح من خلال بعض الامثلة على (OWC) لاستخدام (IFMS) واحدة من نظريات النقطة الصامدة وستتم دراسة شاورد باستخدام العلاقات الضمنية، يمكن اعتبار هذا العمل بيانا عاما للفضاء المترى الضبابي واظهار نوع اخر من نظريات النقطة الصامدة في التخطيط البياني المتوافق الضعيف احيانا والتي يدرسها باحثون اخرون وتوحيد العديد من النتائج في موضوع النقطة الثابتة .

**الكلمات المفتاحية:** الفضاء المترى الضبابي، الفضاء المترى الضبابي الحدسي، نظريات النقطة الصامدة شاورد، اربعة في تخطيط البياني متوافق الضعيف، التخطيط البياني المتوافق الضعيف احيانا.

### 1-Introduction

Atanassove [1] In 1986 insert & the calculated understandable from various fixed point theories at fuzzy sets an intuitionistic as a universal notice to the fuzzy group, which is inserted through Zadeh[2]. We underline studies after ten years later, Kramosil & Michalek insert note from universal notice from fuzzy metric- spaces [3]. Later on, Park, 2004[4] outlined thought from (IFMS) while that assistance from *continuous t\_norm* together *continuous t\_conorms*. After that lots to works were advanced to the theories on newly, in 2006. Alaca et al. [5] introduced of that outlined from (IFMS) through job usage from Intuitionistic Fuzzy sets, Turkoglu [6] et al. show Jungck's [7] mutual metric-space theories with in check from *intuitionistic fuzzy metric space*. The understandable to weakly commuting

Insert the understandable of (OWC) charts, which is more universal then the understandable of Weakly Compatible-Maps at that sheet, us show some Schauder fixed point theories under that condition from sometimes Weakly Compatible-Maps and (IFMS), while prove another sort of this theories an (IFMS) which is considered at that sheet as a supplementary search tagged Schauder fixed point theories an (IFMS), about corollaries of the central effect is given.

### 2. Preliminaries

Several requisite understandable from fuzzy metric space (IFMS) is offered at that part.

#### Definition1. [15]:

A binary operation  $*$ :  $[0, 1] \times [0, 1] \rightarrow [0, 1]$  is a (continuous *t\_norm*) then  $*$  satisfy the following condition:

- a)  $*$  is commutative & associative,
- b)  $*$  is continuous,
- c)  $a * 1 = a$  for all  $a \in [0, 1]$ ,
- d)  $a * b \leq c * d$ ,  $a \leq c$  and  $b \leq d$  to each  $a, b, c, d \in [0, 1]$ .

**Definition2. [16]:** let X becomes a nonempty set. A popularized metric (or D-metric) on X is a function  $D: W * W * W \rightarrow R^+$ , that satisfy the following condition to every  $x, y, z, a \in W$ :

- a)  $D(x, y, z) \geq 0$ ,
- b)  $D(x, y, z) = 0$  if and only if  $x = y = z$ ,

maps an (IFMS) the understandable of weakly compatible- maps are the most general. They do know the notion from Hausdorff topology fuzzy metric-space, [8]. Proved B. D. Pant & Sunny Chauhan [9] common fixed-point theories to (OWC)-Maps in *manger – space*,

*Surendra Singh & Amardeep Singh* [10] (IFMS) while mutual fixed point theories employ (OWC)-Maps self\_maps, Anupama & Arvind [11], fixed point theories an (IFMS) through employ (OWC) - Maps in rational form, Pranjali while Shailesh [12] several mutual fixed point theories in (IFMS) usage the CLRG estate, In 2016, Arun & Zaheer [13], All couples from Compatible -Maps is Weakly-compatible. After that, many writers show fixed point theories employ several mapping in satisfy spaces, Rashmi & Saurabh [14].

c)  $D(x, y, z) = D(p\{x, y, z\})$ , where  $p$  is the change function

d)  $D(x, y, z) \leq D(x, y, a) + D(a, z, z)$ .

That pair  $(X, D)$  is called the popularized metric neither  $D - metric$  space.

**Definition3. [16]** A 4 - tuple  $(X, M_D, *)$  is called  $M - Fuzzy$  Metric Space if  $X$  is a nonempty set,  $*$  is  $M - continuous$   $t - norm$  &  $M$  is Fuzzy subset of  $X * X * X * (0, \infty)$ , satisfy the following condition to every  $x, y, z, a$  on  $X$  &  $t, s > 0$ :

a)  $M_D(x, y, z, t) > 0$ ,

b)  $M_D(x, y, z, t) = 1$  if and only if  $x = y = z$ ,

c)  $M_D(x, y, z, t) = M_D(p\{x, y, z\}, t)$ , wherever  $p$  is a change mapping of  $x, y$  &  $z$ ,

d)  $M_D(x, y, a, t) * M_D(a, z, z, s) \leq M_D(x, y, z, t + s)$ ,

e)  $M_D(x, y, z, *) : \{ (0, \infty) \rightarrow [0, 1] \}$  is a continuous.

**Example1. [17]:** let  $Z = R$  & let:

$$M_D(w, x, p, t) = t / (t + D(w, x, p)), t > 0$$

Wherever:

$$D(w, x, p) = \max \{ |w - x|, |x - p|, |p - w| \} \quad \forall w, x, p \text{ on } Z$$

Then  $(Z, M_D, *)$  is  $M - Fuzzy$  Metric Space.

**Definition4. [15]:** the binary operation  $\diamond: [0, 1] \times [0, 1] \rightarrow [0, 1]$  is a continuous  $t - conorm$  undefined\* satisfy the following condition:

- 1)  $\diamond$  is commutative while associative,
- 2)  $\diamond$  is discontinuous,
- 3)  $a \diamond 0 = a$ , to every  $a$  on  $[0, 1]$ ,
- 4)  $a \diamond b \leq c \diamond d$  &  $a \leq c$  &  $b \leq d$  to each  $a, b, c, d$  on  $[0, 1]$ .

**Lemma1. [4]:** If  $*$  is a continuous  $t - norm$  &  $\diamond$  is a continuous  $t - conorm$

- 1) To all  $a, b$  on  $[0, 1]$ , if  $a > b$ , then  $c, d \in [0, 1]$  satisfy it  $a * c \geq b \diamond a \geq b \diamond d$ ,
- 2) If  $a$  on  $[0, 1]$ , then  $b, c$  on  $[0, 1]$  satisfy it  $b * b \geq a$  &  $a \geq c \diamond c$ .

**Definition5. [18]:** A 5 - tuple  $(X, M, N, *, \diamond)$  is called for become an (IFMS) if  $X$  an arbitrary sets,  $*$  is a continuous  $t - norm$ ,  $\diamond$  is a continuous  $t - conorm$  &  $M, N$  are fuzzy set on  $X^2 \times [0, \infty)$  satisfy the following condition:

- 1)  $M(x, y, t) + N(x, y, t) \leq 1$  for all  $x, y$  on  $X$  &  $t > 0$ ,
- 2)  $M(x, y, 0) = 0$  for all  $x, y$  on  $X$ ,
- 3)  $M(x, y, t) = 1$  for all  $x, y$  on  $X$  &  $t > 0$  if and only if  $x = y$ ,
- 4)  $M(x, y, t) = M(y, x, t)$  for all  $x, y$  on  $X$  &  $t > 0$ ,
- 5)  $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$  for all  $x, y, z$  on  $X$  &  $s, t > 0$ ,
- 6)  $M(x, y, .) : [0, \infty) \rightarrow [0, 1]$  is left continuous, for all  $x, y$  on  $X$ ,
- 7)  $\lim_{x \rightarrow \infty} M(x, y, t) = 1$  for all  $x, y$  on  $X$  &  $t > 0$ ,
- 8)  $N(x, y, .) = 1$  for all  $x, y$  on  $X$ ,
- 9)  $N(x, y, t) = 0$  for all  $x, y$  on  $X$  &  $t > 0$  if and only if  $x = y$ ,
- 10)  $N(x, y, t) = N(y, x, t)$  for all  $x, y$  on  $X$  &  $t > 0$ ,

- 11)  $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$  for all  $x, y, z$  on  $X$  &  $s, t > 0$ ,
- 12)  $N(x, y, \cdot): [0, \infty) \rightarrow [0, 1]$  is right continuous for all  $x, y$  on  $X$ ,
- 13)  $\lim_{t \rightarrow \infty} N(x, y, t) = 0$ , for all  $x, y$  on  $X$ .

An alternative definition of the convergent & Cauchy series at (IFMS) is given next.

**Definition6. [18]:** Let  $(X, M, N, *, \diamond)$  be at (IFMS). Then

- 1) A sequence  $\{x_n\}$  in  $X$  is called for become Cauchy sequence if at every  $t > 0$  &  $p > 0$ .

$$\lim_{n \rightarrow \infty} M(x_{n+p}, x_n, t) = 0 \text{ \& \ } \lim_{n \rightarrow \infty} M(x_{n+p}, x_n, t) = 1$$

- 2) A sequence  $x_n$  in  $X$  is called for become convergent to a point  $x$  on  $X$  if, to each  $t > 0$ ,

$$\lim_{n \rightarrow \infty} M(x_n, x, t) \text{ \& \ } \lim_{n \rightarrow \infty} N(x_n, x, t) = 0.$$

- 3) If all  $(M, N)$  – (Cauchy sequence is convergent), is called for become  $(M, N)$  - Complete.
- 4) If each sequence in  $X$  includes a convergent subsequence, is called to become compact.

**Definition7. [19]:** Let a quasi\_ metric on a group  $Z$  is a mapping of  $d: X^2 \rightarrow R^2$  satisfy the following condition for all  $w, x, p \in Z$ .

- i)  $d(w, x) = 0$ ,
- ii)  $d(w, x) = d(x, w)$ ,
- iii)  $d(w, p) \leq d(w, x) + d(x, p)$ .

**Remark1. [20]:** The (IFMS)  $(X, M, N, *, \diamond)$  &  $M(x, y, *)$  is non – decreasing &  $N(x, y, \diamond)$  is non – increasing for all  $x, y$  on  $X$ .

The following proposition is fundamental in our work and we will use it later

**Proposition1. [20]:** let  $(X, M, N, *, \diamond)$  become the (IFMS), to every  $r$  on  $[0, 1]$ , where introduce  $d_r: X^2 \rightarrow R^+$  as attached:

$$d_r(x, y) = \inf\{t > 0 \mid M(x, y, t) > 1 - r, N(x, y, t) < r\} \dots (1)$$

- 1)  $(X, d_r): r \in (0, 1]$  is a popularizing distance for a quasi- metric people.
- 2) The topology  $\{T(d_r) \in (X, d_r): r \in (0, 1]\}$  agrees together that  $(M, N)$  – Topology  $\in (X, M, N, *, \diamond)$  then,  $d_r$  is compatible symmetric to  $T(M, N)$  }.

### 3. Schauder Fixed Point Theories using (OWC)

We afford a definition from an intuitionistic Schauder fixed point theory (IFM)  $(X, M, N, *, \diamond)$ . The concept of (IFMS) will become defined & calculated using the park. Sadat & park moreover advanced the notion of intuitionistic fuzzy topology together in metric & normed spaces. We insert an intuitionistic fuzzy decrease mapping & show (FP) theories in (IFMS). To that basal concepts and notes, we consider to [3], [9], [20], [21], [22] and [23].

**Definition8. [9]** Let  $(X, A)$  be an (IFMS), a subset for  $(X, A)$  is called for be bounded if to every  $r$  on  $(0, 1)$  there occur  $t > 0$  show it

$$N(x, y, t) < r \text{ \& \ } M(x, y, t) > 1 - r, \text{ to every } x, y \in X.$$

**Definition9. [24]:** Let  $(X, A)$  be an (IFMS) &  $B \subseteq X$  is called to be a closed set in  $(X, A)$  if for every  $r \in (0, 1)$  if & little if for each sequence  $\{X_n\}$  on  $B$  converges to  $x \in X$  (than,  $\lim_{n \rightarrow \infty} M(x_n, x, t) \geq 1 - r$  &  $\lim_{n \rightarrow \infty} N(x_n, x, t) \leq r$ , to every  $t > 0, x \in B$ ).

In what dependent  $A = (X, N1, M1, *, \diamond)$  &  $B = (Y, N2, M2, *, \diamond)$  will denote two (IFMS), wherever  $X$  &  $Y$  are metric-space.

**Definition10. [25]:** Let A & B be two mapping for at (IFMS)  $(X, M, N, *, \diamond)$  into it-self. The maps A & B are called for be compatible if, to every  $t > 0$ ,

$$\lim_{n \rightarrow \infty} M(ABx_n, BAx_n, t) = 1 \ \& \ \lim_{n \rightarrow \infty} N(ABx_n, BAx_n, t) = 0.$$

When  $\{x_n\}$  is a sequence  $\in X$  satisfy it  $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Bx_n = x$ , from a few  $x \in X$ .

**Definition11. [6]:** Two self \_mappings A & B of an intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  is said for become non-compatible if that exists in list one sequence  $\{X_n\}$  show it

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Bx_n = z, \text{ for a few } z \in X \text{ but neither}$$

$\lim_{n \rightarrow \infty} M(ABx_n, BAx_n, t) \neq 1 \ \& \ \lim_{n \rightarrow \infty} N(ABx_n, BAx_n, t) \neq 0$  or the limit does not occur.

**Definition12. [26]:** Let  $(X, M, N, *, \diamond)$  become in an intuitionistic fuzzy metric space. Let A & B become self-maps on X. Let a point x in X is termed a chance point from A & B if  $Ax = Bx$ . Is that condition,  $w = Ax = Bx$  is termed a point from chance from A & B. In1986. Jungck [25] insert the notion of weakly compatible- maps as follows.

**Definition13. [27]:** Two self-mappings A & B for at intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  are called for becoming at pair from self-mapping (A, B) of at intuitionistic fuzzy metric-space  $(X, M, N, *, \diamond)$  is called for becoming weakly-compatible whether they commute at their chance points ( $Ax = Bx$  from a few  $x \in X$ , then  $ABx = BAx$ ). That is simple to view the couple compatible- maps are weakly-compatible but the converse is not true.

**Definition14. [28]:** Two self – mappings A & B from an intuitionistic fuzzy metric-space  $(X, M, N, *, \diamond)$  are called for becoming (OWC) if that is a point  $x \in X$  which is coincidence – point of A & B, what A & B commute.

**Lemma2. [6]:** let  $(X, M, N, *, \diamond)$  become an intuitionistic fuzzy metric-space & from each  $x, y$  in X,  $t > 0$  & if there exists at number  $r \in (0, 1)$   
 $M(x, y, rt) \geq M(x, y, t) \ \& \ N(x, y, rt) \leq N(x, y, t)$ , satisfy  $x = y$ .

As an illustration, consider the following model:

**Example2** Let  $X = [2, 15]$  &  $(X, M, N, *, \diamond)$  become at intuitionistic fuzzy metric\_ space. Define mappings M, N, O & P:  $X \rightarrow X$  through.

$$M(x) = \begin{cases} 2 & \text{if } x = 2 \\ 3 & \text{if } x > 2 \end{cases} \quad O(x) = \begin{cases} x & \text{if } x = 2 \\ 6 & \text{if } x > 2 \end{cases}$$

$$N(x) = \begin{cases} 2 & \text{if } x = 2 \\ 6 & \text{if } 2 < x \leq 5 \end{cases} \quad P(x) = \begin{cases} 2 & \text{if } x = 2 \\ 12 & \text{if } 2 < x < 5 \\ x - 3 & \text{if } x > 5 \end{cases}$$

Then, where determine

$$M(Ax, By, t) = \frac{t}{(t + |x-y|)}, \ N(Ax, By, t) = \frac{|x-y|}{(t - |x-y|)} \text{ to each } x, y \text{ on } X, t > 0$$

$$M(2) = 2 = O(2) \ \& \ N(2) = 2 = P(2)$$

$$M(2) = 2 = P(2) \ \& \ N(2) = 2 = O(2)$$

That is M, O as well as N and Pare (OWC) & (2) is there exist common fixed-point for M, N, O & P.

**Example 3** Let  $R$  become at intuitionistic fuzzy metric-space  $(X, M, N, *, \diamond)$  & introduce  $M, N, O$  &  $P: R \rightarrow R$  through  $OX = 9x, PX = x^3, Mx = 3x^2$  &  $NX = x^3$ , from each  $x \in R$ . the  $OX = PX, MX = NX$ , from all  $x = 0, 3$  then  $O, P, M$  &  $N$  is (OWC) self – maps.

**Remark2** let  $(X, M)$  is fuzzy metric-space &  $K \subset X$ , then  $\bar{K}$  is closed in  $(X, d_r(x, y))$  &  $\bar{K^r} \subset \bar{K} \forall r \in (0, 1)$ , (where  $\bar{K^r}$  symbolizes the closure of  $K$  in  $(X, d_r(x, y))$ ).

**Definition 15** Let  $(X, M, N, *, \diamond)$  is an intuitionistic fuzzy metric space. A continuous-mapping  $W: X' \times X \times [0, 1] \rightarrow X$  is called for be a convex body  $\in X$  if from each  $x, y$  on  $X$  &  $r \in [0, 1], t > 0$

$$[\varphi(u, W(x, y, r), t) \leq \lambda M(u, x, t) + (1 - \lambda) N(u, y, t)]$$

Holds from each  $u \in X$ . the intuitionistic fuzzy metric-space  $(X, M, N, *, \diamond)$  together a convex body is invited in an intuitionistic fuzzy convex metric- space.

**Theorem 1**

Let  $K$  become a non-empty convex, intuitionistic fuzzy compact sub-set an (IFMS) & let  $A, B, S$  &  $T$  become self\_mappings of  $X$ . is satisfied  $r$  & couple  $\{A, S\}$  &  $\{B, T\}$  become (OWC). Whenever stay  $r \in (0, 1)$  satisfy it (1) using equation (2)

$$M(Ax, By, rt) \geq \varnothing(\text{Min}\{M(Ax, Sx, t), (\frac{k_1 M(Sx, Ty, t) + K_2 M(Ax, Ty, t) + k_3 M(Sx, By, t)}{K_1 + K_2 + K_3}), M(By, Ty, t)\})$$

$$N(Ax, By, rt) \leq \varphi(\text{Max}\{N(Ax, Sx, t), (\frac{k_1 N(Sx, Ty, t) + K_2 N(Ax, Ty, t) + k_3 N(Sx, By, t)}{K_1 + K_2 + K_3}), N(By, Ty, t)\})$$

From each  $x, y \in X, t > 0$  &  $k_1, k_2$  &  $k_3 > 0$ , where  $\varnothing, \varphi: [0, 1] \rightarrow [0, 1]$ ,

Satisfy it  $\varnothing(t) > t$  &  $\varphi(t) < t$ , from each  $t \in [0, 1)$  respectively.

There  $A, B, S$  &  $T$  has a common (FP) in  $X$ ,

**Proof:**

Suppose that couple  $\{A, S\}$  &  $\{B, T\}$  become (OWC), also where are points  $x, y$  on  $X$ . satisfy it  $Ax = Sx$  &  $By = Ty$ . Since satisfies (proposition (1) and  $(X, d_r(Ax, By))$  is a metric space. As  $B$  is an intuitionistic fuzzy-compact sub-set of  $X$ ,  $B$  is a compact sub-set of  $(X, d_r(Ax, By))$  by definition (10) and lemma (2) from all  $r \in (0, 1)$ , where  $d_r(Ax, By)$  denotes the  $r$ -metric space of  $X$ . first allow there  $X$  is intuitionistic fuzzy-bounded there from all  $r \in (0, 1), t(r) > 0$  such it

$$N(Ax, By, t(r)) \leq r \quad \& \quad M(Ax, By, t(r)) \geq 1-r \quad \text{from each } x, y \in X$$

Now we demand that  $Ax = By$ . it not, then by (2), when

$$\begin{aligned} M(Ax, By, rt) &\geq \varnothing(\text{Min}\{M(Ax, Sx, t), (\frac{k_1 M(Sx, Ty, t) + K_2 M(Ax, Ty, t) + k_3 M(Sx, By, t)}{K_1 + K_2 + K_3}), M(By, Ty, t)\}) \\ &\geq \varnothing(\text{Min}\{M(Ax, Ax, t(r)), (\frac{k_1 M(Ax, By, t(r)) + K_2 M(Ax, By, t(r)) + k_3 M(Ax, By, t(r))}{K_1 + K_2 + K_3}), M(By, Ty, t)\}) \\ &\geq \varnothing(\text{Min}\{1, (\frac{K_1 + K_2 + K_3}{K_1 + K_2 + K_3}). M(Ax, By, t(r)), 1\}) \\ &\geq \varnothing(\text{Min}\{1, 1. (M(Ax, By, t(r)), 1\}) \end{aligned}$$

$$\geq \emptyset M(Ax, By, t(r))$$

$$M(Ax, By, rt) \geq 1 - r$$

&

$$N(Ax, By, rt) \leq \varphi(\text{Max}\{N(Ax, Sx, t), (\frac{k_1 N(Sx, Ty, t) + K_2 N(Ax, Ty, t) + k_3 N(Sx, By, t)}{K_1 + K_2 + K_3}),$$

$$N(By, Ty, t)\}$$

$$\leq \varphi(\text{Max}\{N(Ax, Ax, t), (\frac{k_1 N(Ax, By, t(r)) + K_2 N(Ax, By, t(r)) + k_3 N(Ax, By, t(r))}{K_1 + K_2 + K_3}), N$$

$$(By, By, t(r))\}$$

$$N(Ax, By, rt) \leq \varphi(\text{Max}\{0, (\frac{k_1 + K_2 + k_3}{K_1 + K_2 + K_3}) \cdot N(Ax, By, t(r)), 0\}$$

$$\leq \varphi(\text{Max}\{(0, 1 \cdot N(Ax, By, t(r)), 0\}$$

$$N(Ax, By, rt) \leq \varphi(N(Ax, By, t(r))) \leq r$$

$$N(Ax, By, rt) \leq r$$

First, from the definition of bounded where

$$[dr(Ax, By)] = \inf\{t > 0: N(Ax, +By, t(r)) < r \ \& \ M(Ax, By, t(r)) > 1 - r\} \ r \in (0,1) \text{by} \dots \dots (1)$$

From proposition (1) we have  $dr(Ax, By) \leq t(r)$  from each  $x, y \in X$  such it  $X$  is bounded with respect to  $dr(Ax, By)$

Conversely, suppose there  $X$  is bounded together esteem to  $dr(Ax, By), 0 < r < 1$ . Then from all  $r \in (0, 1)$  there exists  $t(r)$ , such it:

$$dr(Ax, By) \leq t(r) \text{ if } x, y \in X \text{ that is } dr(Ax, By) \leq t(r) < t(r) + 1, \text{ then } x, y \in X.$$

$M(Ax, By, t(r) + 1) > 1 - r, N(Ax, By, t(r) + 1) < r$  from each  $x, y \in X$  Show their  $X$  is intuitionistic fuzzy- bounded.

Second, suppose that  $B$  is intuitionistic fuzzy-closed. Take  $r_0 \in (0, 1)$ . let  $\{X_n\}$  be a sequence in  $B$  such that  $\lim_{n \rightarrow \infty} d_{r_0}(x_n, x) = 0$ . Now to a offered  $\epsilon > 0$ , that exist a positive -integer  $N(\epsilon)$  show it  $d_{r_0}(X_n, X) < \epsilon$ , from each  $n \geq N(\epsilon)$  by eq. (1)

$$M(Ax_n, Bx, r \epsilon) \geq \emptyset(\text{Min}\{M(Ax_n, Ax, \epsilon), (\frac{k_1 M(x_n, Bx, \epsilon) + K_2 M(x_n, Bx, \epsilon) + k_3 M(x_n, Bx, \epsilon)}{K_1 + K_2 + K_3}),$$

$$M(Bx_n, Bx, \epsilon)\}$$

$$M(Ax_n, Bx, r \epsilon) \geq \emptyset(\text{Min}\{1, (\frac{k_1 + K_2 + k_3}{K_1 + K_2 + K_3}) \cdot M(Ax_n, Bx, \epsilon), 1\})$$

$$M(Ax_n, Bx, r \epsilon) \geq \emptyset(\text{Min}\{1, 1 \cdot M(Ax_n, Bx, \epsilon), 1\})$$

$$M(Ax_n, Bx, r \epsilon) \geq \emptyset M(Ax_n, Bx, \epsilon) \geq 1 - r_0, \lim_{n \rightarrow \infty} M(Ax_n, Bx, r \epsilon) \geq 1 - r_0$$

$$N(Ax_n, Bx, r \epsilon) \leq \varphi(\text{Max}\{N(Ax_n, Ax, \epsilon), (\frac{k_1 N(x_n, Bx, \epsilon) + K_2 N(x_n, Bx, \epsilon) + k_3 N(x_n, Bx, \epsilon)}{K_1 + K_2 + K_3}),$$

$$N(Bx_n, Bx, \epsilon)\}$$



$$N(Ax_n, Bx, r) \leq \varphi(\text{Max}\{1, (\frac{k_1 + K_2 + k_3}{K_1 + K_2 + K_3}) \cdot N(Ax_n, Bx, \epsilon), 1\})$$

$$N(Ax_n, Bx, r) \leq \varphi(\text{Max}\{1, 1 \cdot N(Ax_n, Bx, \epsilon), 1\})$$

$$N(Ax_n, Bx, r) \leq \varphi N(Ax_n, Bx, \epsilon) \leq r_0, \lim_{n \rightarrow \infty} N(Ax_n, Bx, r) \leq r_0$$

Then  $x \in B$  such that is closed with respect to  $d_{r_0}(x_n, x)$ . since  $0 < r_0 < 1$  is spot, at next it  $B$  is closed together esteem for  $d_r(x_n, x)$ ,  $0 < r_0 < 1$ .

Converse following by conditions (1), (6) & (12) of definition (5),

Third, assume it  $B$  intuitionistic fuzzy-compact take  $r_0 \in (0, 1)$  let  $\{X_n\}$  become a sequence in  $B$ . thus their exist a sub-sequence  $\{X_{n_k}\}$  &  $x$  in  $B$  (both depending on  $r_0$ ) such that

$\lim_{k \rightarrow \infty} M(x_{n_k}, x, t) \geq 1 - r_0$  while  $\lim_{k \rightarrow \infty} N(x_{n_k}, x, t) \leq r_0$ , for all  $t > 0$ . For a given  $\epsilon > 0$  with  $r_0 - \epsilon > 0$  & to  $t > 0$  that occur a positive integer  $K(\epsilon, t)$  show it by eq. (1)  $Ax_{n_k} = Bx$

$$M(Ax_{n_k}, Bx, rt) \geq \varphi(\text{Min}\{M(Ax_{n_k}, Ax, t), (\frac{k_1 M(Ax_{n_k}, Bx, t) + K_2 M(Ax_{n_k}, Bx, t) + k_3 M(Ax_{n_k}, Bx, t)}{K_1 + K_2 + K_3})\}, M(Bx_{n_k}, Bx, t))$$

$$M(Ax_{n_k}, Bx, rt) \geq \varphi(\text{Min}\{1, (\frac{k_1 + K_2 + k_3}{K_1 + K_2 + K_3}) \cdot M(Ax_{n_k}, Bx, t), 1\})$$

$$M(Ax_{n_k}, Bx, rt) \geq \varphi(\text{Min}\{1, 1 \cdot M(Ax_{n_k}, Bx, t), 1\})$$

$$M(Ax_{n_k}, Bx, rt) \geq \varphi M(Ax_{n_k}, Bx, t) \geq 1 - r_0 + \epsilon$$

$$\lim_{k \rightarrow \infty} M(Ax_{n_k}, Bx, rt) \geq 1 - r_0 + \epsilon$$

$$N(Ax_{n_k}, Bx, rt) \leq \varphi(\text{Max}\{N(Ax_{n_k}, Ax, t), (\frac{k_1 N(Ax_{n_k}, Bx, t) + K_2 N(Ax_{n_k}, Bx, t) + k_3 N(Ax_{n_k}, Bx, t)}{K_1 + K_2 + K_3})\}, N(Bx_{n_k}, Bx, t))$$

$$N(Ax_{n_k}, Bx, rt) \leq \varphi(\text{Max}\{1, (\frac{k_1 + K_2 + k_3}{K_1 + K_2 + K_3}) \cdot N(Ax_{n_k}, Bx, t), 1\})$$

$$N(Ax_{n_k}, Bx, rt) \leq \varphi(\text{Max}\{1, 1 \cdot N(Ax_{n_k}, Bx, t), 1\})$$

$$N(Ax_{n_k}, Bx, rt) \leq \varphi N(Ax_{n_k}, Bx, t) \leq r_0 - \epsilon$$

$$\lim_{k \rightarrow \infty} N(Ax_{n_k}, Bx, rt) \leq r_0 - \epsilon$$

$$\lim_{k \rightarrow \infty} d_{r_0 - \epsilon}(x_{n_k}, x) = 0$$

Then  $B$  is compact with respect to  $d_{r_0 - \epsilon}(x_{n_k}, x)$  since  $r_0 \in (0, 1)$  &  $\epsilon > 0$  are arbitrary  $x_{n_k}$  it follows at  $B$  is compact with respect to  $d(x_{n_k}, x)$  from all  $r \in (0, 1)$

The converse follows for bounded & closed, there  $T$  has existed common (FP) hence it follow up it  $K$  is a non-empty convex & compact sub-group for the metric space  $(X, d_{r_0}(Ax, By))$ , so by Schauder fixed point theories [29] (let  $A$  become a closed convex sub-group of a Banach space & supposing that occurs a continuous-map  $T$  sending  $A$  to countably compact sub-set  $T(A)$  for  $A$  than  $T$  has (FP)) it follows it  $T$  has a Schauder constant dot theories.



**Theorem (2)**

Let  $K$  becomes a non- empty intuitionistic fuzzy closed convex subgroup of at (IFMS) with  $\bar{T}(k)$  being intuitionistic fuzzy-compact & letting  $A, B, S$  &  $T$  become person-charts for  $X$ .

Let couple  $\{A, S\}$  &  $\{B, T\}$  become (OWC). undefined that occur  $r \in (0, 1)$  satisfy it (1) by eq. (2), from each  $x, y \in X, t > 0$  &  $k_1, k_2, k_3 > 0$ , when  $\emptyset, \varphi: [0,1] \rightarrow [0,1]$  satisfy that  $\emptyset(t) > t$  &  $\varphi(t) < t$  from each  $t \in [0, 1)$  respectively. When  $A, B, S$  &  $T$  there exist common (FP)  $\in X$ .

**Proof:**

Assume it the two  $\{A, S\}$  &  $\{B, T\}$  become (OWC), also where are points  $x, y$  on  $X$ . satisfy it

$$Ax = Sy \text{ \& } By = Ty.$$

Since satisfies proposition (1), then from theorem (1) that  $(X, d_r(Ax, By))$  is a metric space.

A gain since  $K$  is intuitionistic fuzzy-closed,  $K$  is closed with esteem to  $d_r(Ax, By)$  from all  $r \in (0, 1)$ .

Also since  $\bar{T}(k)$  is intuitionistic fuzzy- compact by theorem (1).

$\bar{T}(K)$  is compact with respect to  $d_r(Ax, By)$  from all  $r \in (0, 1)$ .

In particular  $\bar{T}(k)$  is compact an  $(X, d_{r_0}(Ax, Bx))$ .

As well for remark (2)  $\bar{T}(k)$  is closed in  $(X, d(Ax, By)_{r_0})$  &  $\bar{T}(k)^{r_0} \subset \bar{T}(k)$

Where  $\bar{T}(k)^{r_0}$  is the closure of  $T(K)$  in  $(X, d(Ax, By)_{r_0})$  so  $\bar{T}(k)^{r_0}$  is compact in  $(X, d(Ax, By)_{r_0})$ .

Thus  $K$  is a non-empty closed convex sub-group for (FMS)  $(X, d(Ax, By)_r)$ . Therefore so by Schauder fixed point theories [29], it follows up it  $T$  has (FP) theories.

**4. Conclusion:** we extend the concept of any one of the types of fixed point in intuitionistic fuzzy metric space by occasionally weakly compatible of self-mapping and its results are a generalization of fixed point theories through the use of the intuitionistic fuzzy metric space.

**Recommendation:** we recommend the use of any one of the types of fixed point in intuitionistic fuzzy cone metric space and use our results of fixed point theories through the use of the intuitionistic fuzzy metric space.

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